Day 32

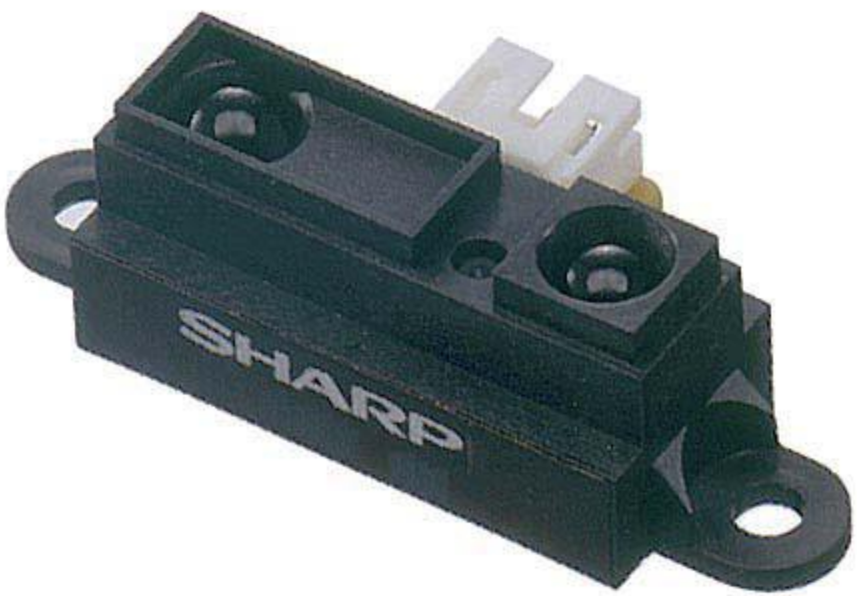
Range Sensor Models

Range Sensors

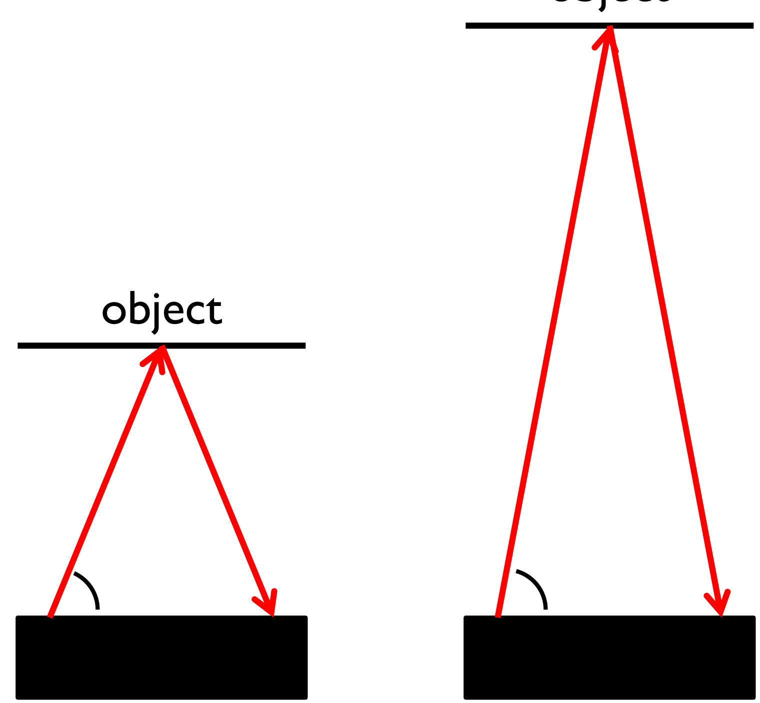
- range sensors measure the distance between the robot and the sensed object(s)
  - bearing measurement can be obtained by rotating the sensor or using multiple sensors arranged on a circular arc
- many non-vision based sensors can be modeled by measuring the distance along a beam or cone

Infra-red Range Sensor

- uses an IR LED and a linear CCD array to triangulate distances



Sharp GP2Y0A21YK0F data sheet



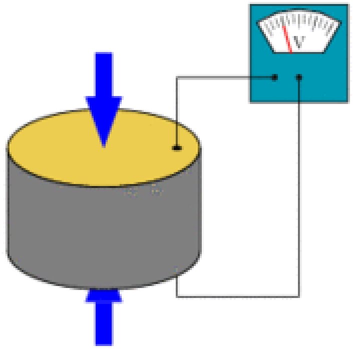
angle increases as distance increases

Infra-red Range Sensor

- has a minimum and maximum distance
  - changes depending on the model but range is on the order of a few centimeters to a few meters
- inexpensive (less than $20)
- measurements affected by the material properties of the object
  - but less sensitive to the orientation of the reflecting surface
- low accuracy

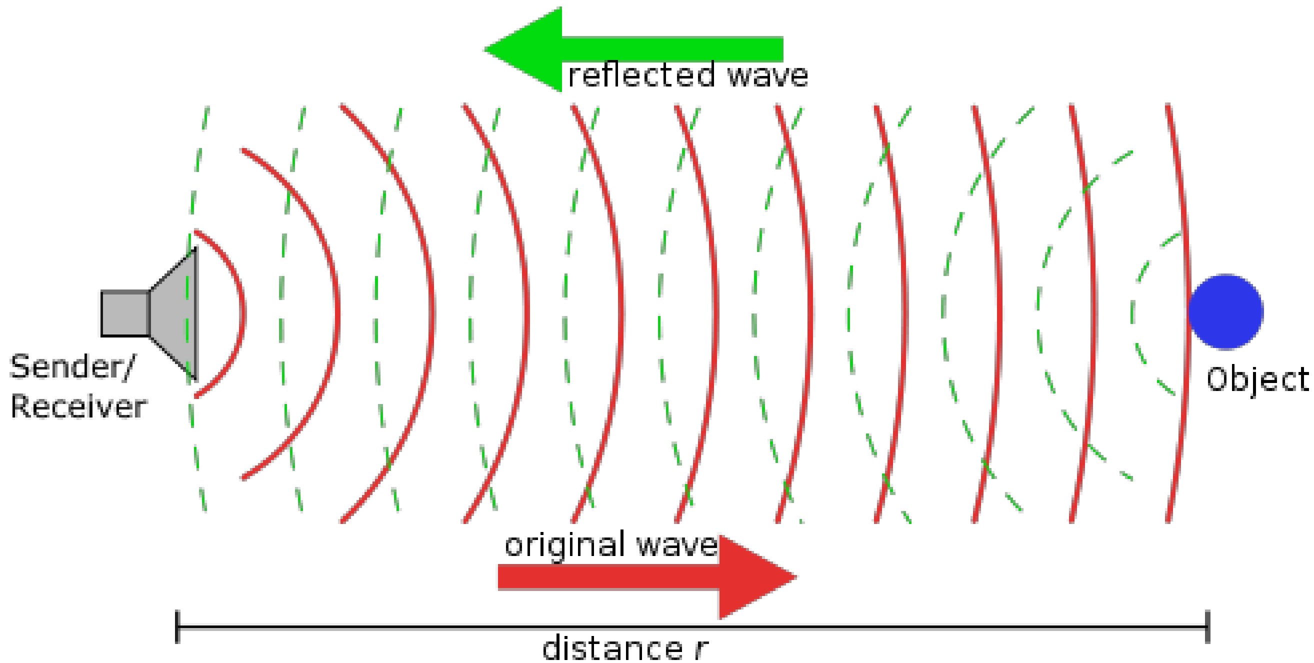
Ultrasound Range Sensors

- ultrasound is sound above the upper limit of human hearing (approximately 20,000 Hz)
- inexpensive ultrasonic transducers are essentially ceramic discs that vibrate at ultrasonic frequencies
  - operate via the piezoelectric effect
    - an applied external electric field causes a small change in the physical dimensions
    - conversely, an applied stress will induce a small electric field



Ultrasound Range Sensors

- basic idea of operation is simple
  - transducer emits a short pulse of ultrasound
  - receiver listens for echo
  - time elapsed between pulse and echo is proportional to twice the distance to the object



Ultrasound Range Sensors

- speed of sound in air is given approximately by

$$c = 331 + 0.6T \ \frac{\text{m}}{\text{s}}$$

  where T is temperature in Celsius
  - humidity affects c
- modern electronics can easily measure elapsed time with sufficient precision to obtain good distance resolution

Ultrasound Range Sensors

- most materials are specular reflectors of ultrasound

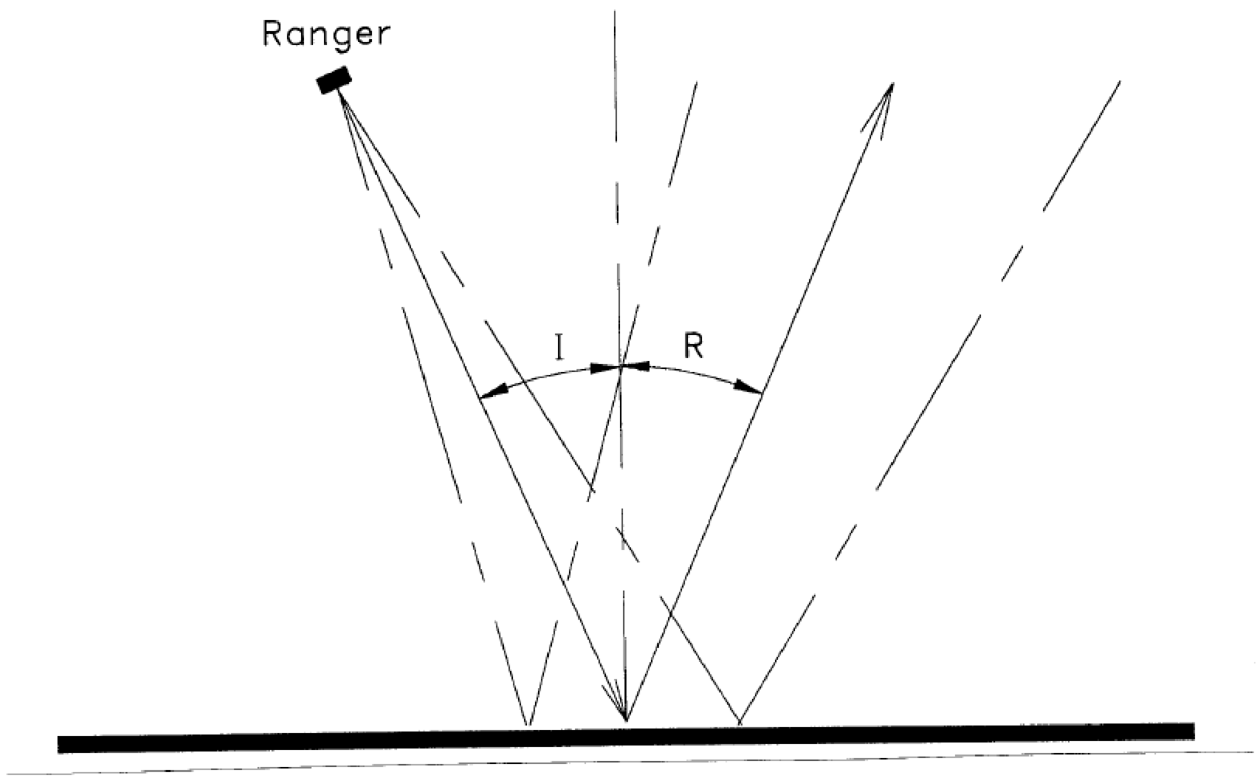


Figure 3.—Angle of incidence. (I = angle of incidence, R = angle of reflection.)

Ultrasound Range Sensors

- multiple reflections produce erroneous distance measurements

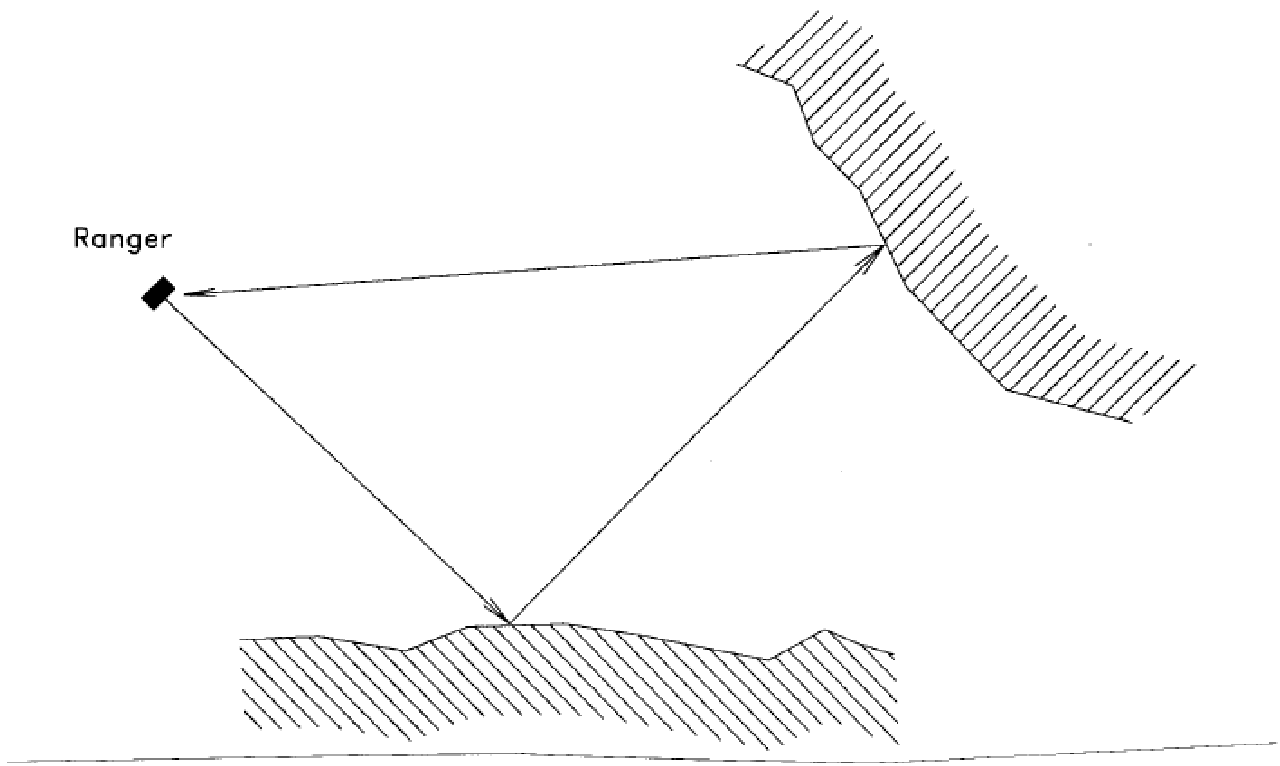


Figure 4.—Multiple reflections causing false range reading.

Ultrasound Range Sensors

- unfocussed ultrasound transducers have a wide beam pattern with significant side lobes

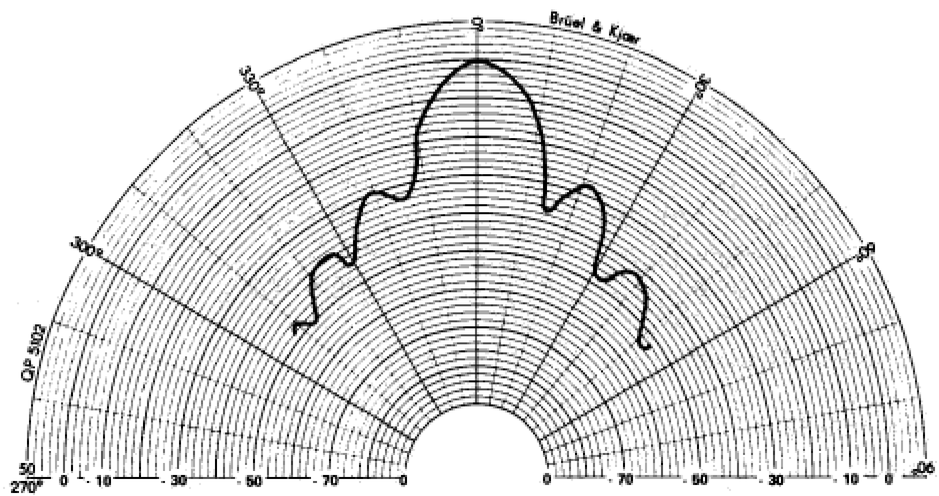


Figure 2: Typical propagation pattern for the Polaroid 6500 Series ultrasonic sensor. Courtesy of Polaroid [1].

Cao and Borenstein, "Experimental Characterization of Polaroid Ultrasonic Sensors in Single and Phased Array Configuration"

Ultrasound Range Sensors

- beamwidth + side lobes leads to spurious measurements

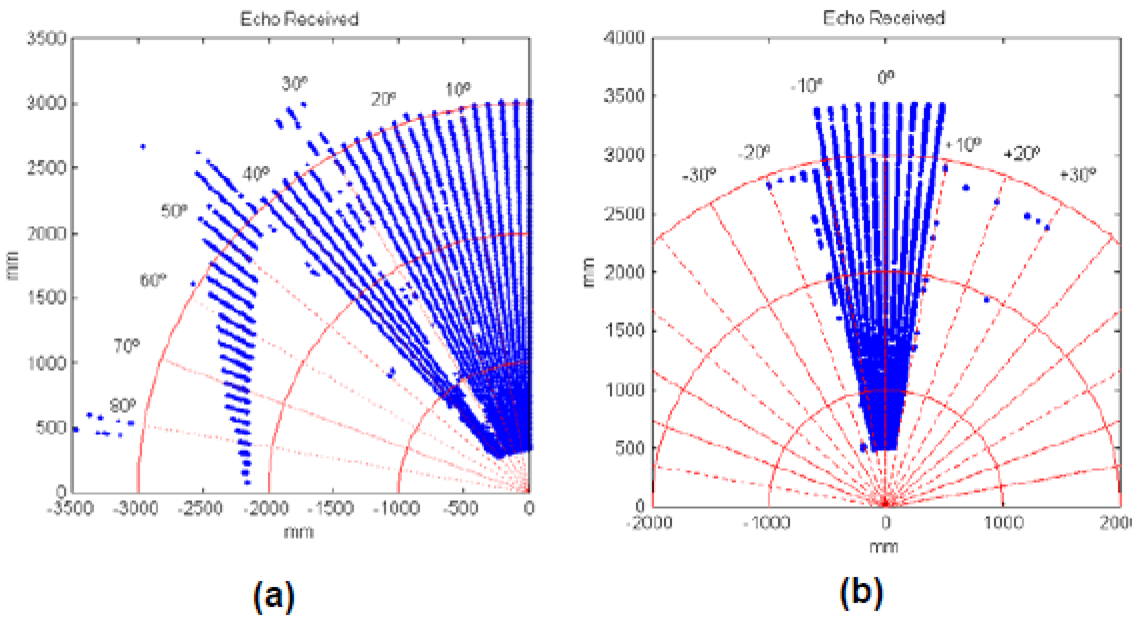
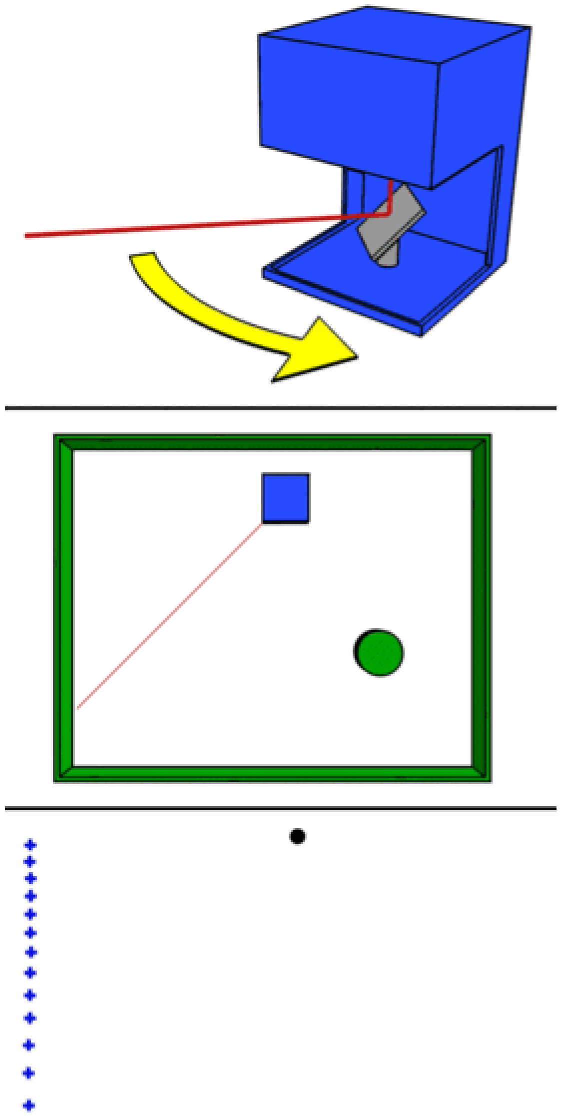


Figure 4: Baseline results for measuring distance to (a) the board and (b) the pole placed at different distances and with the sonar paned to different angles between 0 and 90°.

Cao and Borenstein, "Experimental Characterization of Polaroid Ultrasonic Sensors in Single and Phased Array Configuration"

LIDAR

- light detection and ranging
- [commercial range finder disassembled]



LIDAR

- for time of flight sensor $d = \frac{1}{2} c \Delta t$
  - speed of light $c = 3 \times 10^8 \ \frac{\text{m}}{\text{s}}$

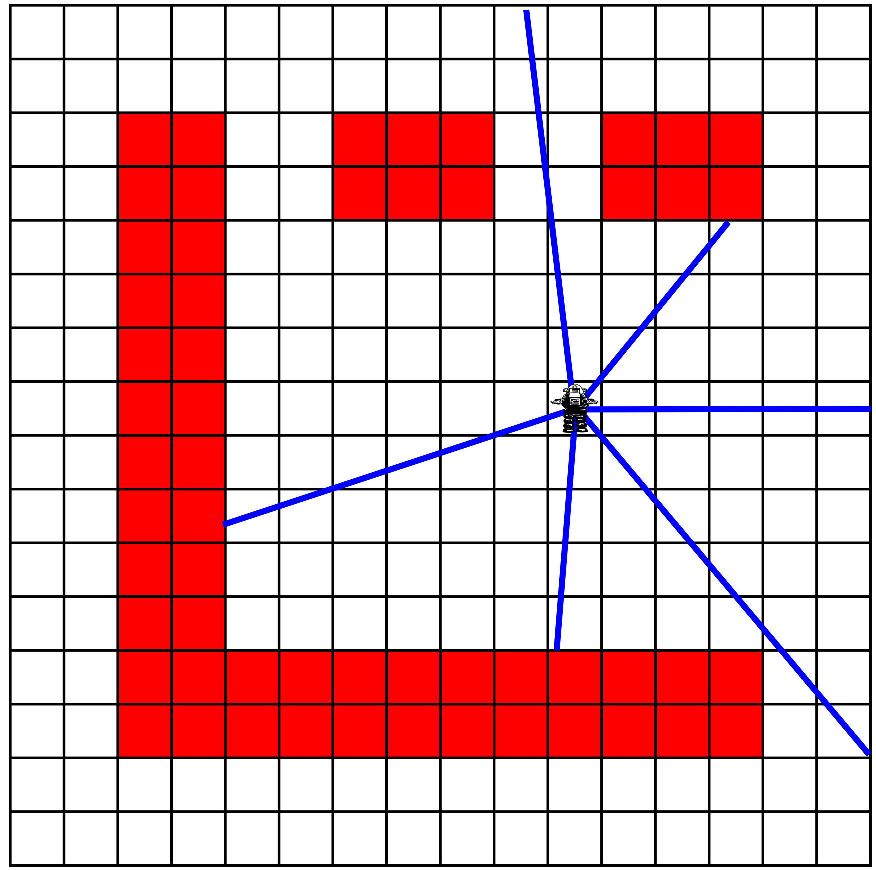
    $$
    \begin{aligned}
    1 \text{ ns} &= 150 \text{ mm} \\
    100 \text{ ps} &= 15 \text{ mm} \\
    10 \text{ ps} &= 1.5 \text{ mm}
    \end{aligned}
    $$

- typical angular resolution below 1 degree
- typical errors on the order of 10 cm or less
- cost > $1,000

Beam Models of Range Finders

- given the map and the robot's location, find the probability density that the range finder detects an object at a distance z_t^k along a beam

$$p(z_t^k \mid x_t, m)$$



the k is here because range sensors typically return many measurements at one time; e.g., the sensor might return a full 360 degree scan made up of K measurements at once

$$z_t = \left\{ z_t^1, z_t^2, \ldots, z_t^k, \ldots, z_t^K \right\}$$

Beam Models of Range Finders

- most range finders have a minimum and maximum range
- we seek a model that can represent
  1. the correct range measurement with noise
  2. unexpected obstacles
  3. failures
  4. random measurements

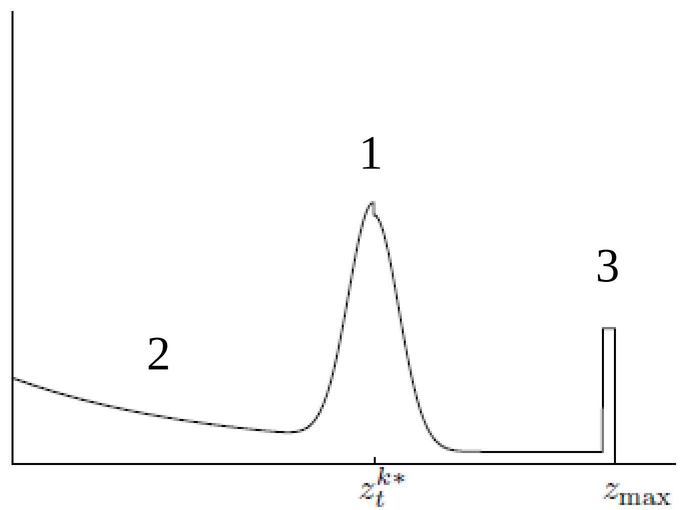


Figure 6.4 "Pseudo-density" of a typical mixture distribution $p(z_t^k \mid x_t, m)$.

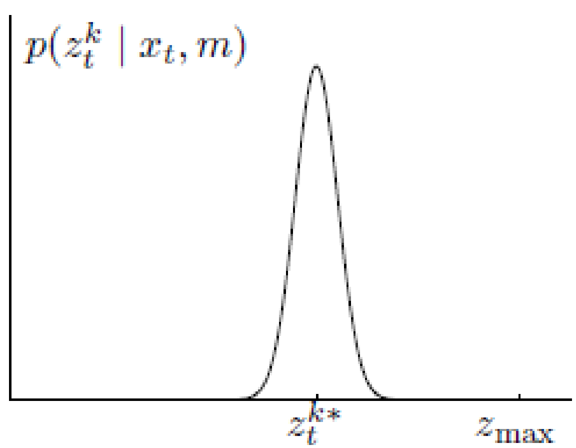
Beam Models of Range Finders

1. Correct Measurement with Noise

- suppose that the beam intersects an obstacle at a range of z_t^{k*}
  - model this measurement as a narrow Gaussian with mean z_t^{k*}

$$p_{\text{hit}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, N\left(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2\right) & \text{if } 0 \le z_t^k \le z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

(a) Gaussian distribution p_{hit}



Beam Models of Range Finders

2. Unexpected Obstacles

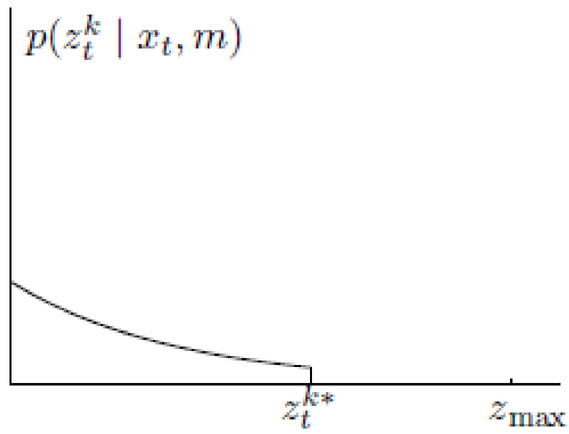
- in a dynamic environment there will be obstacles not found on a static map; these obstacles will cause range measurements shorter than expected
- if such obstacles appear continuously and independently at a constant average rate then they can be modeled as a Poisson process
- the time between events in a Poisson process has an exponential distribution with probability density function

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Beam Models of Range Finders

$$p_{\text{short}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, \lambda_{\text{short}} e^{-\lambda_{\text{short}} z_t^k} & \text{if } 0 \le z_t^k \le z_t^{k*} \\ 0 & \text{otherwise} \end{cases}$$

(b) Exponential distribution p_{short}



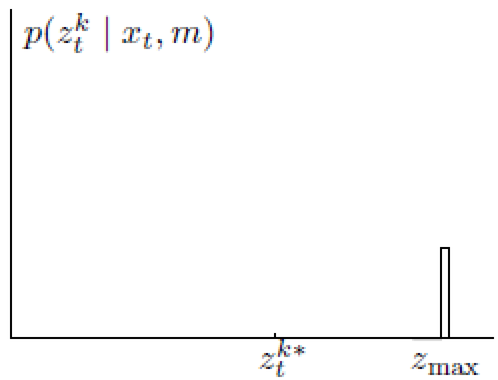
Beam Models of Range Finders

3. Failures

- range finders can fail to sense an obstacle in which case most sensors return $z_{\max}$
- modeled as a point mass distribution centered at $z_{\max}$

$$p_{\max}(z_t^k \mid x_t, m) = \begin{cases} 1 & \text{if } z_t^k = z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

(c) Uniform distribution $p_{\max}$

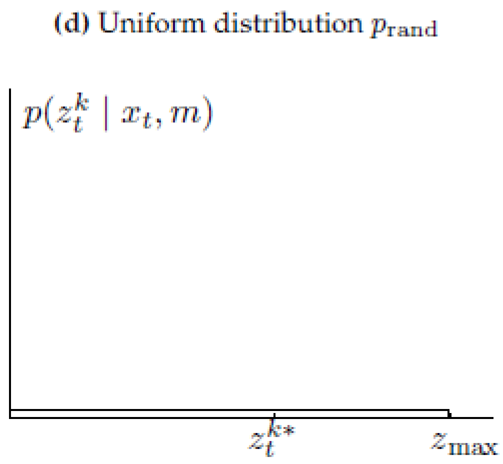


Beam Models of Range Finders

4. Random Measurements

- unexplainable measurements are modeled as a uniform distribution over the range $[0, z_{\max}]$

$$p_{\text{rand}}(z_t^k \mid x_t, m) = \begin{cases} \frac{1}{z_{\max}} & \text{if } 0 \le z_t^k \le z_{\max} \\ 0 & \text{otherwise} \end{cases}$$



Beam Models of Range Finders

- the complete model is a weighted sum of the previous four densities with weights

$$w_{\text{hit}} + w_{\text{short}} + w_{\max} + w_{\text{rand}} = 1$$

$$p(z_t^k \mid x_t, m) = \begin{bmatrix} w_{\text{hit}} \\ w_{\text{short}} \\ w_{\max} \\ w_{\text{rand}} \end{bmatrix}^T \begin{bmatrix} p_{\text{hit}}(z_t^k \mid x_t, m) \\ p_{\text{short}}(z_t^k \mid x_t, m) \\ p_{\max}(z_t^k \mid x_t, m) \\ p_{\text{rand}}(z_t^k \mid x_t, m) \end{bmatrix}$$